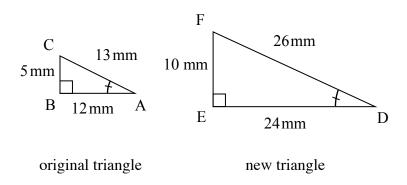
SCALING FIGURES AND SCALE FACTOR

Geometric figures can be reduced or enlarged. When this change happens, every length of the figure is reduced or enlarged equally (proportionally), and the measures of the corresponding angles stay the same.

The ratio of any two corresponding sides of the original and new figure is called a scale factor. The scale factor may be written as a percent or a fraction. It is common to write new figure measurements over their original figure measurements in a scale ratio, that is, $\frac{\text{NEW}}{\text{ORIGINAL}}$.

For additional information, see the Math Notes box in Lesson 4.1.2 of the *Core Connections*, *Course 2* text.

Example 1 using a 200% enlargement



Side length ratios:

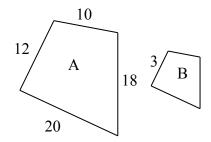
DE AB	=	$\frac{24}{12}$	=	$\frac{2}{1}$
FD CA	=	$\frac{26}{13}$	=	$\frac{2}{1}$
FE CB	=	$\frac{10}{5}$	=	$\frac{2}{1}$

The scale factor for length is 2 to 1.

Example 2

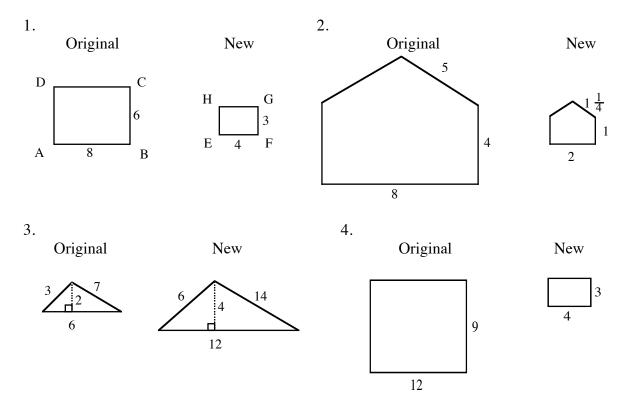
Figures A and B at right are similar. Assuming that Figure A is the original figure, find the scale factor and find the lengths of the missing sides of Figure B.

The scale factor is $\frac{3}{12} = \frac{1}{4}$. The lengths of the missing sides of Figure B are: $\frac{1}{4}(10) = 2.5$, $\frac{1}{4}(18) = 4.5$, and $\frac{1}{4}(20) = 5$.



Problems

Determine the scale factor for each pair of similar figures in problems 1 through 4.



- 5. A triangle has sides 5, 12, and 13. The triangle was enlarged by a scale factor of 300%.
 - a. What are the lengths of the sides of the new triangle?
 - b. What is the ratio of the perimeter of the new triangle to the perimeter of the original triangle?
- 6. A rectangle has a length of 60 cm and a width of 40 cm. The rectangle was reduced by a scale factor of 25%.
 - a. What are the dimensions of the new rectangle?
 - b. What is the ratio of the perimeter of the new rectangle to the perimeter of the original rectangle?

1.
$$\frac{4}{8} = \frac{1}{2}$$
 2. $\frac{2}{8} = \frac{1}{4}$

 3. $\frac{2}{1}$
 4. $\frac{1}{3}$

 5. a. 15, 36, 39 b. $\frac{3}{1}$
 6. a. 15 cm and 10 cm b. $\frac{1}{4}$

PROPORTIONAL RELATIONSHIPS

A **proportion** is an equation stating the two ratios (fractions) are equal. Two values are in a proportional relationship if a proportion may be set up to relate the values.

For more information, see the Math Notes boxes in Lessons 4.2.3, 4.2.4, and 7.2.2 of the *Core Connections*, *Course* 2 text. For additional examples and practice, see the *Core Connections*, *Course* 2 Checkpoint 9 materials.

Example 1

The average cost of a pair of designer jeans has increased \$15 in 4 years. What is the unit growth rate (dollars per year)?

Solution: The growth rate is $\frac{15 \text{ dollars}}{4 \text{ years}}$. To create a unit rate we need a denominator of "one." $\frac{15 \text{ dollars}}{4 \text{ years}} = \frac{x \text{ dollars}}{1 \text{ year}}$. Using a Giant One: $\frac{15 \text{ dollars}}{4 \text{ years}} = 4 \frac{4}{1 \text{ year}} \div \frac{x \text{ dollars}}{1 \text{ year}} \Rightarrow 3.75 \frac{\text{dollars}}{\text{year}}$.

Example 2

Ryan's famous chili recipe uses 3 tablespoons of chili powder for 5 servings. How many tablespoons are needed for the family reunion needing 40 servings?

Solution: The rate is $\frac{3 \text{ tablespoons}}{5 \text{ servings}}$ so the problem may be written as a proportion: $\frac{3}{5} = \frac{t}{40}$.

One method of solving the proportion is to use the Giant One:

Another method is to use **cross multiplication**:

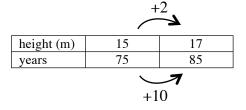
is to use the Giant One:	$\frac{3}{5} = \frac{t}{40}$
$\frac{3}{5} = \frac{t}{40} \Longrightarrow \frac{3}{5} \cdot \boxed{\frac{8}{8}} = \frac{24}{40} \Longrightarrow t = 24$	$\frac{3}{5}$ \times $\frac{t}{40}$
5 40 5 28 40	$5 \cdot t = 3 \cdot 40$
	5t = 120
	t = 24

Finally, since the unit rate is $\frac{3}{5}$ tablespoon per serving, the equation $t = \frac{3}{5}s$ represents the general proportional situation and one could substitute the number of servings needed into the equation: $t = \frac{3}{5} \cdot 40 = 24$. Using any method the answer is 24 tablespoons.

Example 3

Based on the table at right, what is the unit growth rate (meters per year)?

Solution: $\frac{2 \text{ meters}}{10 \text{ years}} = \frac{2 \text{ meters} \cdot \frac{1}{10}}{10 \text{ years} \cdot \frac{1}{10}} = \frac{\frac{1}{5} \text{ meter}}{1 \text{ year}} = \frac{1}{5} \frac{\text{meter}}{\text{ year}}$



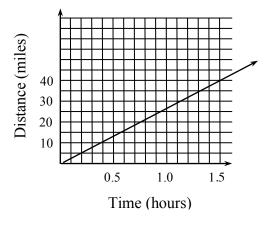
Problems

For problems 1 through 10 find the unit rate. For problems 11 through 25, solve each problem.

- 1. Typing 731 words in 17 minutes (words per minute)
- 2. Reading 258 pages in 86 minutes (pages per minute)
- 3. Buying 15 boxes of cereal for \$43.35 (cost per box)
- 4. Scoring 98 points in a 40 minute game (points per minute)
- 5. Buying $2\frac{1}{4}$ pounds of bananas cost \$1.89 (cost per pound)
- 6. Buying $\frac{2}{3}$ pounds of peanuts for \$2.25 (cost per pound)
- 7. Mowing $1\frac{1}{2}$ acres of lawn in $\frac{3}{4}$ of a hour (acres per hour)
- 8. Paying \$3.89 for 1.7 pounds of chicken (cost per pound)

9	weight (g)	6	8	12	20
	length	15	20	30	50
	(cm)				

- 10. For the graph at right, what is the rate in miles per hour?
- 11. If a box of 100 pencils costs \$4.75, what should you expect to pay for 225 pencils?
- 12. When Amber does her math homework, she finishes 10 problems every 7 minutes. How long will it take for her to complete 35 problems?



- 13. Ben and his friends are having a TV marathon, and after 4 hours they have watched 5 episodes of the show. About how long will it take to complete the season, which has 24 episodes?
- 14. The tax on a \$600 vase is \$54. What should be the tax on a \$1700 vase?

- 15. Use the table at right to determine how long it will take the Spirit club to wax 60 cars.
- 16. While baking, Evan discovered a recipe that required $\frac{1}{2}$ cups of walnuts for every $2\frac{1}{4}$ cups of flour. How many cups of walnuts will he need for 4 cups of flour?
- 17. Based on the graph, what would the cost to refill 50 bottles?
- 18. Sam grew $1\frac{3}{4}$ inches in $4\frac{1}{2}$ months. How much should he grow in one year?
- 19. On his afternoon jog, Chris took 42 minutes to run $3\frac{3}{4}$ miles. How many miles can he run in 60 minutes?
- ars.cars waxed816cipe that
 $2\frac{1}{4}$ cups of
ill he needhours3640353040353030cost to refill2540How much1540
 - 10 5 2 4 6 8 10 12 bottles refilled

32

12

- 20. If Caitlin needs $1\frac{1}{3}$ cans of paint for each room in her house, how many cans of paint will she need to paint the 7-room house?
- 21. Stephen receives 20 minutes of video game time every 45 minutes of dog walking he does. If he wants 90 minutes of game time, how many hours will he need to work?
- 22. Sarah's grape vine grew 15 inches in 6 weeks, write an equation to represent its growth after *t* weeks.
- 23. On average Max makes 45 out of 60 shots with the basketball, write an equation to represent the average number of shots made out of x attempts.
- 24. Write an equation to represent the situation in problem 14 above.
- 25. Write an equation to represent the situation in problem 17 above.

1.	$43 \frac{\text{words}}{\text{minute}}$	2.	$3 \frac{\text{pages}}{\text{minute}}$	3.	$2.89 \frac{\$}{box}$	4.	$2.45 \frac{\text{points}}{\text{minute}}$
5.	$0.84 \frac{\$}{\text{pound}}$	6.	3.38 ^{\$} / _{pound}	7.	2 <u>acre</u> hour	8.	$2.29 \frac{\$}{pound}$
9.	$\frac{2}{5}$ grams centimeter	10.	$\approx 27 \frac{\text{miles}}{\text{hour}}$	11.	\$10.69	12.	24.5 min.
13.	19.2 hours	14.	\$153	15.	22.5 hours	16.	$\frac{8}{9}$ cup
17.	\$175	18.	$4\frac{2}{3}$ inches	19.	≈ 5.36 miles	20.	$9\frac{1}{3}$ cans
21.	$3\frac{3}{8}$ hours	22.	$g = \frac{5}{2}t$	23.	$s = \frac{3}{4}x$	24.	t = 0.09c
25.	C = 3.5b						

RATES AND UNIT RATES

Rate of change is a ratio that describes how one quantity is changing with respect to another. Unit rate is a rate that compares the change in one quantity to a one-unit change in another quantity. Some examples of rates are miles per hour and price per pound. If 16 ounces of flour cost \$0.80 then the unit cost, that is the cost per one once, is $\frac{\$0.80}{16} = \0.05 .

For additional information see the Math Notes box in Lesson 4.2.4 of the *Core Connections*, *Course 2* text. For additional examples and practice, see the *Core Connections*, *Course 2* Checkpoint 9 materials.

Example 1

A rice recipe uses 6 cups of rice for 15 people. At the same rate, how much rice is needed for 40 people?

The rate is: $\frac{6 \text{ cups}}{15 \text{ people}}$ so we need to solve $\frac{6}{15} = \frac{x}{40}$.

The multiplier needed for the Giant One is $\frac{40}{15}$ or $2\frac{2}{3}$.

Using that multiplier yields $\frac{6}{15} \cdot \frac{2\frac{2}{3}}{2\frac{2}{3}} = \frac{16}{40}$ so 16 cups of rice is needed.

Note that the equation $\frac{6}{15} = \frac{x}{40}$ can also be solved using proportions.

Example 2

Arrange these rates from least to greatest:

30 miles in 25 minutes 60 miles in one hour 70 miles in $1\frac{2}{3}$ hr

Changing each rate to a common denominator of 60 minutes yields:

 $\frac{30 \text{ mi}}{25 \text{ min}} = \frac{x}{60} \Longrightarrow \frac{30}{25} \cdot \frac{2.4}{2.4} = \frac{72}{60} \frac{\text{mi}}{\text{min}} \qquad \frac{60 \text{ mi}}{1 \text{ hr}} = \frac{60 \text{ mi}}{60 \text{ min}} \qquad \frac{70 \text{ mi}}{1\frac{2}{3} \text{ hr}} = \frac{70 \text{ mi}}{100 \text{ min}} = \frac{x}{60} \Longrightarrow \frac{70}{100} \cdot \frac{0.6}{0.6} = \frac{42 \text{ mi}}{60 \text{ min}}$

So the order from least to greatest is: 70 miles in $1\frac{2}{3}$ hr < 60 miles in one hour < 30 miles in 25 minutes. Note that by using 60 minutes (one hour) for the common unit to compare speeds, we can express each rate as a unit rate: 42 mph, 60 mph, and 72 mph.

Example 3

A train in France traveled 932 miles in 5 hours. What is the unit rate in miles per hour?

Unit rate means the denominator needs to be 1 hour so: $\frac{932 \text{ mi}}{5 \text{ hr}} = \frac{x}{1 \text{ hr}}$. Solving by using a Giant One of $\frac{0.2}{0.2}$ or simple division yields x = 186.4 miles per hour.

Problems

Solve each rate problem below. Explain your method.

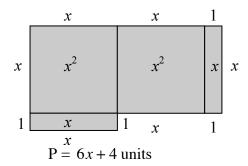
- 1. Balvina knows that 6 cups of rice will make enough Spanish rice to feed 15 people. She needs to know how many cups of rice are needed to feed 135 people.
- 2. Elaine can plant 6 flowers in 15 minutes. How long will it take her to plant 30 flowers at the same rate?
- 3. A plane travels 3400 miles in 8 hours. How far would it travel in 6 hours at this rate?
- 4. Shane rode his bike for 2 hours and traveled 12 miles. At this rate, how long would it take him to travel 22 miles?
- 5. Selina's car used 15.6 gallons of gas to go 234 miles. At this rate, how many gallons would it take her to go 480 miles?
- 6. Arrange these readers from fastest to slowest: Abel read 50 pages in 45 minutes, Brian read 90 pages in 75 minutes, and Charlie read 175 pages in 2 hours.
- Arrange these lunch buyers from greatest to least assuming they buy lunch 5 days per week: Alice spends \$3 per day, Betty spends \$25 every two weeks, and Cindy spends \$75 per month.
- 8. A train in Japan can travel 813.5 miles in 5 hours. Find the unit rate in miles per hour.
- 9. An ice skater covered 1500 meters in 106 seconds. Find his unit rate in meters per second.
- 10. A cellular company offers a price of \$19.95 for 200 minutes. Find the unit rate in cost per minute.
- 11. A car traveled 200 miles on 8 gallons of gas. Find the unit rate of miles per gallon and the unit rate of gallons per mile.
- 12. Lee's paper clip chain is 32 feet long. He is going to add paper clips continually for the next eight hours. At the end of eight hours the chain is 80 feet long. Find the unit rate of growth in feet per hour.

1.	54 cups	2.	75 min	3.	2550 miles	4.	$3\frac{2}{3}$ hr
5.	32 gallons	6.	C, B, A	7.	C, A, B	8.	162.7 mi/hr
9.	≈14.15 m/s	10.	≈ \$0.10/min	11.	25 m/g; $\frac{1}{25}$ g/m	12.	6 ft/hr

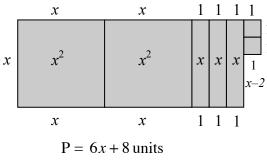
ALGEBRA TILES AND PERIMETER

Algebraic expressions can be represented by the perimeters of algebra tiles (rectangles and squares) and combinations of algebra tiles. The dimensions of each tile are shown along its sides and the tile is named by its area as shown on the tile itself in the figures at right. When using the tiles, perimeter is the distance around the exterior of a figure.

Example 1







х

x

 x^2

x

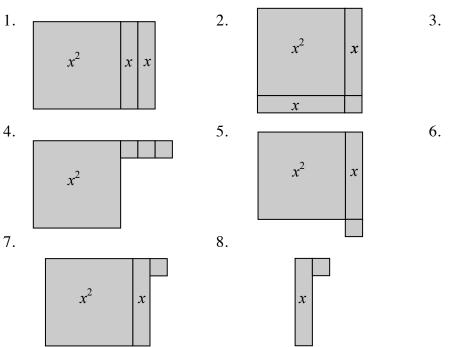
x

 x^2

Problems

50

Determine the perimeter of each figure.





1

х

1

1

x

x

1.	4x + 4 un.	2.	4x + 4 un.	3.	2x + 8 un.	4.	4x + 6 un.
5.	4x + 4 un.	6.	4x + 2 un.	7.	4x + 4 un.	8.	2x + 4 un.

COMBINING LIKE TERMS

Algebraic expressions can also be simplified by combining (adding or subtracting) terms that have the same variable(s) raised to the same powers, into one term. The skill of combining like terms is necessary for solving equations. For additional information, see the Math Notes box in Lesson 4.3.2 of the *Core Connections, Course 2* text. For additional examples and practice, see the *Core Connections, Course 2* Checkpoint 7A materials.

Example 1

Combine like terms to simplify the expression 3x + 5x + 7x.

All these terms have x as the variable, so they are combined into one term, 15x.

Example 2

Simplify the expression 3x + 12 + 7x + 5.

The terms with x can be combined. The terms without variables (the constants) can also be combined.

3x + 12 + 7x + 5	
3x + 7x + 12 + 5	Note that in the simplified form the term with the variable is listed before the constant term.
10x + 17	

Example 3

Simplify the expression $5x + 4x^2 + 10 + 2x^2 + 2x - 6 + x - 1$.

 $5x + 4x^{2} + 10 + 2x^{2} + 2x - 6 + x - 1$ $4x^{2} + 2x^{2} + 5x + 2x + x + 10 - 6 - 1$ $6x^{2} + 8x + 3$

Note that terms with the same variable but with different exponents are not combined and are listed in order of decreasing power of the variable, in simplified form, with the constant term last.

Example 4

The algebra tiles, as shown in the Perimeter Using Algebra Tiles section, are used to model how to combine like terms.

The large square represents x^2 , the rectangle represents x, and the small square represents one. We can only combine tiles that are alike: large squares with large squares, rectangles with rectangles, and small squares with small squares. If we want to combine: $2x^2 + 3x + 4$ and $3x^2 + 5x + 7$, visualize the tiles to help combine the like terms:

 $2x^2$ (2 large squares) + 3x (3 rectangles) + 4 (4 small squares) + $3x^2$ (3 large squares) + 5x (5 rectangles) + 7 (7 small squares)

The combination of the two sets of tiles, written algebraically, is: $5x^2 + 8x + 11$.

Example 5

Sometimes it is helpful to take an expression that is written horizontally, circle the terms with their signs, and rewrite *like* terms in vertical columns before you combine them:

$$(2x^{2} - 5x + 6) + (3x^{2} + 4x - 9)$$

$$(2x^{2} - 5x + 6) + (3x^{2} + 4x - 9)$$

$$2x^{2} - 5x + 6$$

$$+ 3x^{2} + 4x - 9$$

$$(4x^{2} - 5x + 6)$$

$$(5x^{2} - x - 3)$$
This product is the each the each

procedure may make it easier to tify the terms as well as the sign of term.

Problems

Combine the following sets of terms.

1. $(2x^2 + 6x + 10) + (4x^2 + 2x + 3)$

3.
$$(8x^2 + 3) + (4x^2 + 5x + 4)$$

5.
$$(4x^2 - 7x + 3) + (2x^2 - 2x - 5)$$

7.
$$(5x+6) + (-5x^2 + 6x - 2)$$

- 9. $3c^2 + 4c + 7x 12 + (-4c^2) + 9 6x$
- 2. $(3x^2 + x + 4) + (x^2 + 4x + 7)$
- 4. $(4x^2 + 6x + 5) (3x^2 + 2x + 4)$
- 6. $(3x^2 7x) (x^2 + 3x 9)$
- 8. $2x^2 + 3x + x^2 + 4x 3x^2 + 2$
- 10. $2a^2 + 3a^3 4a^2 + 6a + 12 4a + 2$

- 1. $6x^2 + 8x + 13$ 2. $4x^2 + 5x + 11$ 3. $12x^2 + 5x + 7$ 4. $x^2 + 4x + 1$ 5. $6x^2 - 9x - 2$ 6. $2x^2 - 10x + 9$ 7. $-5x^2 + 11x + 4$ 8. 7x + 2
- $-c^{2}+4c+x-3$ 10. $3a^{3}-2a^{2}+2a+14$ 9

DISTRIBUTIVE PROPERTY

The Distributive Property shows how to express sums and products in two ways: a(b+c) = ab + ac. This can also be written (b+c)a = ab + ac.

Factored form	Distributed form	Simplified form
a(b+c)	a(b) + a(c)	ab + ac

To simplify: Multiply each term on the inside of the parentheses by the term on the outside. Combine terms if possible.

For additional information, see the Math Notes box in Lesson 4.3.3 of the *Core Connections*, *Course 2* text.

Example 1	Example 2	Example 3
2(47) = 2(40 + 7)	$3(x+4) = (3 \cdot x) + (3 \cdot 4)$	$4(x+3y+1) = (4 \cdot x) + (4 \cdot 3y) + 4(1)$
$= (2 \cdot 40) + (2 \cdot 7)$	= 3x + 12	=4x+12y+4
= 80 + 14 = 94		

Problems

Simplify each expression below by applying the Distributive Property.

1.	6(9 + 4)	2.	4(9+8)	3.	7(8+6)	4.	5(7 + 4)
5.	3(27) = 3(20 + 7)	6.	6(46) = 6(40+6)	7.	8(43)	8.	6(78)
9.	3(x+6)	10.	5(x+7)	11.	8(x-4)	12.	6(x-10)
13.	(8+x)4	14.	(2+x)5	15.	-7(x+1)	16.	-4(y+3)
17.	-3(y-5)	18.	-5(b-4)	19.	-(x+6)	20.	-(x+7)
21.	-(x-4)	22.	-(-x-3)	23.	x(x + 3)	24.	4x(x+2)
25.	-x(5x-7)	26.	-x(2x-6)				

Answers

1.	$(6 \cdot 9) + (6 \cdot 4) = 54$	+ 24	= 78	2.	$(4\cdot 9) + (4\cdot 8)$	= 36 -	+ 32 = 68
3.	56 + 42 = 98	4.	35 + 20 = 55	5.	60 + 21 = 81	6.	240 + 36 = 276
7.	320 + 24 = 344	8.	420 + 48 = 468	9.	3x + 18	10.	5x + 35
11.	8x - 32	12.	6x - 60	13.	4x + 32	14.	5x + 10
15.	-7x - 7	16.	-4y - 12	17.	-3y + 15	18.	-5b + 20
19.	-x - 6	20.	-x - 7	21.	-x + 4	22.	<i>x</i> + 3
23.	$x^2 + 3x$	24.	$4x^2 + 8x$	25.	$-5x^2 + 7x$	26.	$-2x^2 + 6x$

When the Distributive Property is used to reverse, it is called factoring. Factoring changes a sum of terms (no parentheses) to a product (with parentheses.)

ab + ac = a(b + c)

To factor: Write the common factor of all the terms outside of the parentheses. Place the remaining factors of each of the original terms inside of the parentheses.

Example 4	Example 5	Example 6
$4x + 8 = 4 \cdot x + 4 \cdot 2$	$6x^2 - 9x = 3x \cdot 2x - 3x \cdot 3$	$6x + 12y + 3 = 3 \cdot 2x + 3 \cdot 4y + 3 \cdot 1$
=4(x+2)	= 3x(2x-3)	= 3(2x+4y+1)

Problems

Factor each expression below by using the Distributive Property in reverse.

1.	6 <i>x</i> +12	2.	5 <i>y</i> -10	3.	8x + 20z	4.	$x^2 + xy$
5.	8 <i>m</i> +24	6.	16y + 40	7.	8 <i>m</i> -2	8.	25 <i>y</i> -10
9.	$2x^2 - 10x$	10.	$21x^2 - 63$	11.	$21x^2 - 63x$	12.	15 <i>y</i> +35
13.	4x + 4y + 4z	14.	6x + 12y + 6	15.	$14x^2 - 49x + 28$	16.	$x^2 - x + xy$

1.	6(x+2)	2.	5(y-2)	3.	4(2x+5z)	4.	x(x+y)
5.	8(<i>m</i> +3)	6.	8(2y+5)	7.	4(2m-1)	8.	5(5y-2)
9.	2x(x-5)	10.	$21(x^2 - 3)$	11.	21x(x-3)	12.	5(3y+7)
13.	4(x+y+z)	14.	6(x+2y+1)	15.	$7(2x^2 - 7x + 4)$	16.	x(x-1+y)

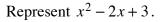
Single Region Expression Mats

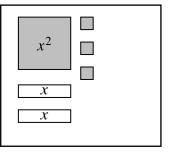
Algebra tiles and Expression Mats are concrete organizational tools used to represent algebraic expressions. Pairs of Expression Mats can be modified to make Expression Comparison Mats (see next section) and Equation Mats. Positive tiles are shaded and negative tiles are blank. A matching pair of tiles with one tile shaded and the other one blank represents zero (0).

= +1

 $\Box = -1$

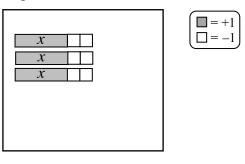
Example 1





Example 2

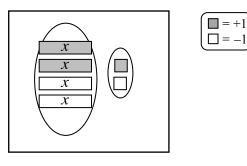
Represent 3(x-2).



Note that 3(x-2) = 3x - 6.

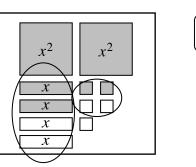
Example 3

This expression makes zero.



Example 4

Simplify $2x^2 + 2x + 2 + (-2x) + (-3)$.



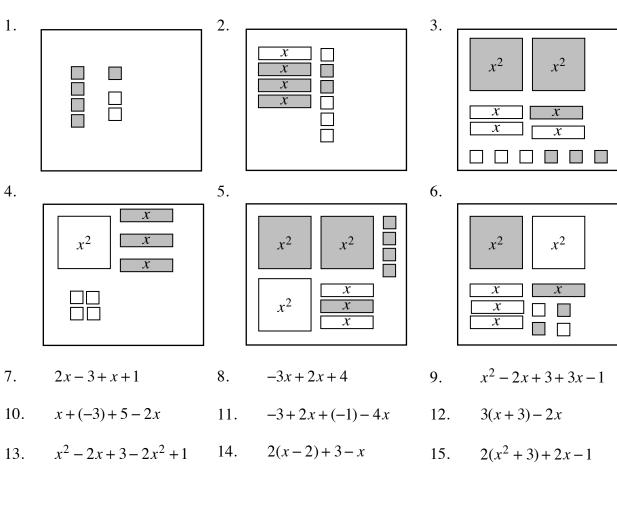
After removing zeros, $2x^2 - 1$ remains.

= +1

 $\Box = -1$

Problems

Simplify each expression.



Answers

1.	3	2.	2x - 2	3.	$2x^2 - 2x$
4.	$-x^2 + 3x - 4$	5.	$x^2 - x + 4$	6.	-2x
7.	3x - 2	8.	-x + 4	9.	$x^2 + x + 2$
10.	-x + 2	11.	-2x - 4	12.	x + 9
13.	$-x^2 - 2x + 4$	14.	x-1	15.	$2x^2 + 2x + 5$

 $\square = +1$ $\square = -1$